Correlation functions of higher-dimensional Luttinger liquids

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Using higher-dimensional bosonization, we study correlation functions of fermions with singular forward scattering. Following Bares and Wen [Phys. Rev. B 48, 8636 (1993)], we consider density-density interactions in d dimensions that diverge in Fourier space for small momentum transfers $|\mathbf{q}|$ as $|\mathbf{q}|^{-\eta}$ with $\eta = 2(d-1)$. In this case the single-particle Green's function shows Luttinger liquid behavior. However, in contrast to $d = 1$, in higher dimensions the singularity of the momentum distribution $n_{\mathbf{k}}$ for wave-vectors $\mathbf{k}$ close to the Fermi wave-vector k_F is characterized by a *different exponent* than the singularity of the density of states $\rho(\omega)$ for frequencies ω close to zero. We also calculate the irreducible polarization $\Pi_\star(\mathbf{q},\omega)$ for $|\mathbf{q}| \approx 2k_F$. Whereas in the one-dimensional Tomonaga-Luttinger model $\Pi_\star(\pm 2k_F + q, \omega)$ exhibits anomalous scaling, we show that in $d > 1$ the leading singular corrections cancel. Thus, even though singular density-density interactions in $d > 1$ lead to Luttinger liquid behavior of the single-particle Green's function, gauge invariant correlation functions such as the polarization or the conductivity show conventional Fermi liquid behavior. We discuss consequences for the effect of disorder on higher-dimensional Luttinger liquids.

I. INTRODUCTION

In spite of intense theoretical efforts, the question whether correlated electrons in two dimensions can exhibit a non-Fermi liquid (NFL) ground state still has no generally accepted answer. The solution to this problem may be the key to a proper understanding of superconductivity in the cuprates. There is little doubt that all experiments probing the normal state of the optimally doped cuprates can be consistently interpreted in terms of NFL behavior [1]. However, from the theoretical side there exists no agreement on the microscopic mechanism responsible for the experimentally seen NFL behavior. The difficulty lies in the fact that a possible breakdown of the Fermi liquid state cannot be described by treating the electron-electron interactions perturbatively. Therefore, the development of non-perturbative methods in dimensions $d > 1$ is rather important.

Recently, considerable progress has been made for the special case that the interaction between the electrons is dominated by forward scattering. This means that the typical momentum $\mathbf{q}$ transfered by the interaction has a magnitude that is small compared with the Fermi wave-vector k_F. In this case the single-particle Green's function $G(\mathbf{k},\omega)$ satisfies an asymptotic Ward-identity, which can be used to sum the most singular terms in the expansion of $G(\mathbf{k},\omega)$ to all orders in the interaction [2]. This Ward identity forms also the basis [3] for the generalization of the bosonization method to arbitrary dimensions [3–7].

In this work we shall use higher-dimensional bosonization to study one- and two-particle correlation functions of higher-dimensional Luttinger liquids. As a microscopic model system for a NFL in $d > 1$, we shall consider spinless fermions interacting with a long-range density-density interaction $V(\mathbf{r}-\mathbf{r}')$, such that the Fourier transform $f_{\mathbf{q}}$ of the interaction is

$$f_{\mathbf{q}} = \frac{g_c^2}{|\mathbf{q}|^\eta} e^{-|\mathbf{q}|/q_c} \; . \tag{1}$$

Here g_c^2 is some coupling constant and $q_c \ll k_F$ is an ultraviolet cutoff. Interactions of this type have been studied by Bares and Wen [8], who showed that for $\eta = 2(d-1)$ the quasi-particle residue vanishes logarithmically at the Fermi surface. The single-particle Green's function exhibits then anomalous scaling, similar to the one-dimensional Tomonaga-Luttinger model. In the regime $\eta < 2(d-1)$ the quasi-particle residue remains finite, and for $\eta > 2(d-1)$ the singularities are even stronger than logarithmic. Note that $\eta \geq d$ corresponds to an interaction $V(\mathbf{r}-\mathbf{r}')$ that diverges for $|\mathbf{r}-\mathbf{r}'| \to \infty$. For simplicity, we shall therefore restrict ourselves in this work to $\eta < d$ [9]. At the first sight long-range interactions of this type seem to be irrelevant to real physical systems. Note, however, that strong correlations between electrons can in some cases be described in terms of long-range gauge forces. In fact, recently several authors [10] have used gauge field theories to explain the experimentally observed non-Fermi liquid behavior in the normal state of the high-temperature superconductors [1]. As already pointed out by Bares and Wen [8], the careful analysis of the long-range interactions of the type (1) might also shed some light onto the nature of the Luttinger liquid state due the coupling to gauge fields in $d = 2$. Our approach is complementary to the philosophy adopted in Refs. [11,12], where a model Green's function of the Luttinger liquid type in $d > 1$ is assumed without microscopic derivation. In this paper we shall show that not all of the well known properties of one-dimensional Luttinger liquids survive in $d > 1$. As discussed in Sec.IV, this has important consequences for the effect of impurities on higher-dimensional Luttinger liquids.

II. THE SINGLE-PARTICLE GREEN'S FUNCTION

To begin with, let us briefly outline the calculation of the single-particle Green's function of interacting fermions with dominant forward scattering within higher-dimensional bosonization [3–7]. First of all, the Fermi surface is subdivided into a finite number M of small patches with diameter Λ such that $q_c \ll \Lambda \ll k_F$. Denoting by $\mathbf{k}^\alpha$, $\alpha = 1, \ldots, M$ a set of wave-vectors on the Fermi surface pointing to the centers of the patches, the non-interacting energy dispersion $\epsilon_{\mathbf{k}}$ is locally expanded $\epsilon_{\mathbf{k}^\alpha+\mathbf{q}} - \mu \approx \mathbf{v}^\alpha \cdot \mathbf{q} + \mathbf{q}^2/2m^\alpha$, where $\mu = \epsilon_{\mathbf{k}^\alpha}$. In the simplest approximation, the inverse effective mass $1/m^\alpha$ is set equal to zero. By doing this we neglect the curvature of the Fermi surface, so that the patches are completely flat. Following Refs. [3,5–7,13], we represent correlation functions as imaginary time Grassmannian functional integrals and decouple the two-body interaction by means of Hubbard-Stratonovich fields $V^\alpha(\mathbf{q}, i\omega_m)$ (where $\omega_m = 2\pi m/\beta$ is a bosonic Matsubara frequency, β is the inverse temperature). Integrating over the fermions, we finally arrive at the following expression for the real-space imaginary-time Green's function

$$G(\mathbf{r}-\mathbf{r}', \tau-\tau') = \sum_{\alpha=1}^{M} e^{i\mathbf{k}^\alpha\cdot(\mathbf{r}-\mathbf{r}')} \langle \mathcal{G}^\alpha(\mathbf{r}, \mathbf{r}', \tau, \tau')\rangle \ . \quad (2)$$

Here $\langle \ldots \rangle$ denotes averaging with respect to the effective action of the Hubbard-Stratonovich field, and $\mathcal{G}^\alpha$ is the solution of

$$\left[-\partial_\tau + i\mathbf{v}^\alpha\cdot\nabla_{\mathbf{r}} + \frac{\nabla_{\mathbf{r}}^2}{2m^\alpha} - V^\alpha(\mathbf{r},\tau)\right]\mathcal{G}^\alpha(\mathbf{r},\mathbf{r}',\tau,\tau') = \delta(\mathbf{r}-\mathbf{r}')\delta^*(\tau-\tau') \ , \quad (3)$$

where $V^\alpha(\mathbf{r},\tau) = \sum_{\mathbf{q}\omega_m} e^{i(\mathbf{q}\cdot\mathbf{r}-\omega_m\tau)} V^\alpha(\mathbf{q}, i\omega_m)$, and $\delta^*(\tau-\tau')$ is the anti-periodic imaginary-time δ-function. In the limit $1/m^\alpha \to 0$ (i.e. for linearized energy dispersion) Eq.(3) can be solved exactly [16,3,5,13], with the result

$$\mathcal{G}^\alpha(\mathbf{r},\mathbf{r}',\tau,\tau') = G_0^\alpha(\mathbf{r}-\mathbf{r}',\tau-\tau') e^{\Phi^\alpha(\mathbf{r},\tau)-\Phi^\alpha(\mathbf{r}',\tau')} \ , \quad (4)$$

where $G_0^\alpha(\mathbf{r},\tau)$ is the solution of Eq.(3) for $V^\alpha = 0$, and

$$\Phi^\alpha(\mathbf{r},\tau) = \sum_{\mathbf{q}\omega_m} \frac{e^{i(\mathbf{q}\cdot\mathbf{r}-\omega_m\tau)}}{i\omega_m - \mathbf{v}^\alpha\cdot\mathbf{q}} V^\alpha(\mathbf{q}, i\omega_m) \ . \quad (5)$$

Furthermore, for $1/m^\alpha \to 0$ the closed loop theorem [2,3,14] justifies the Gaussian approximation for the averaging in Eq.(2), so that we obtain

$$G(\mathbf{r},\tau) = \sum_{\alpha=1}^{M} e^{i\mathbf{k}^\alpha\cdot\mathbf{r}} G_0^\alpha(\mathbf{r},\tau) e^{Q^{\alpha\alpha}(\mathbf{r},\tau)} \ , \quad (6)$$

where for later reference we have introduced a Debye-Waller factor $Q^{\alpha\alpha'}(\mathbf{r},\tau)$ with two patch indices α and α',

$$Q^{\alpha\alpha'}(\mathbf{r},\tau) = S^{\alpha\alpha'}(0,0) - S^{\alpha\alpha'}(\mathbf{r},\tau) \ , \quad (7)$$

$$S^{\alpha\alpha'}(\mathbf{r},\tau) = \frac{1}{\beta\mathcal{V}} \sum_{\mathbf{q}\omega_m} \frac{f^{\rm RPA}(\mathbf{q}, i\omega_m)\cos(\mathbf{q}\cdot\mathbf{r}-\omega_m\tau)}{[i\omega_m - \mathbf{v}^\alpha\cdot\mathbf{q}][i\omega_m - \mathbf{v}^{\alpha'}\cdot\mathbf{q}]} \ . \quad (8)$$

Here $\mathcal{V}$ is the volume of the system, and

$$f^{\rm RPA}(\mathbf{q}, i\omega_m) = \frac{f_{\mathbf{q}}}{1 + f_{\mathbf{q}}\Pi_0(\mathbf{q}, i\omega_m)} \quad (9)$$

is the screened interaction within the random-phase approximation (RPA).

It should be mentioned that a priori it is not clear whether it is allowed to linearize the energy dispersion (corresponding to taking the limit $1/m^\alpha \to 0$) or not. As discussed in Ref. [3], in dimensions $d > 1$ this approximation becomes more questionable than in $d = 1$, because for momenta $\mathbf{q}$ parallel to the Fermi surface the linear term $\mathbf{v}^\alpha\cdot\mathbf{q}$ vanishes, so that the quadratic term $\mathbf{q}^2/2m^\alpha$ is the leading one. In Refs. [3,6] a systematic method for including into the bosonization procedure the effects associated with the finiteness of $1/m^\alpha$ has been developed. Although the expressions derived in these works have so far not been analyzed in detail, it can be shown [3] that for the calculation of the momentum distribution in the presence of the singular density-density interactions discussed here it is indeed sufficient to work with linearized energy dispersion.

If the system is a Fermi liquid, then the quasi-particle residue for wave-vectors close to $\mathbf{k}^\alpha$ can be identified with [3,5]

$$Z^\alpha = e^{S^{\alpha\alpha}(0,0)} \ . \quad (10)$$

On the other hand, non-Fermi liquid behavior in the single-particle Green's function manifests itself in the divergence of $S^{\alpha\alpha}(0,0)$ in the limit $\mathcal{V}\to\infty$ and $\beta\to\infty$. In fact, this quantity is closely related to the leading non-trivial vertex correction shown in Fig.1,

$$\Lambda_1(\mathbf{k}, i\tilde\omega_n; \mathbf{q}, i\omega_m) = -\frac{1}{\beta\mathcal{V}} \sum_{\mathbf{q}'\omega_{m'}} f^{\rm RPA}(\mathbf{q}', i\omega_{m'}) \times G_0(\mathbf{k}+\mathbf{q}', i\tilde\omega_{n+m'}) G_0(\mathbf{k}+\mathbf{q}'+\mathbf{q}, i\tilde\omega_{n+m'+m}) \ , \quad (11)$$

where $\tilde\omega_n = 2\pi(n+\frac{1}{2})/\beta$, and $G_0(\mathbf{k}, i\tilde\omega_n) = [i\tilde\omega_n - \epsilon_{\mathbf{k}} + \mu]^{-1}$ is the non-interacting Matsubara Green's function. Noting that for linearized energy dispersion

$$G_0(\mathbf{k}^\alpha + \mathbf{q}, i\tilde\omega_n) = \frac{1}{i\tilde\omega_n - \mathbf{v}^\alpha\cdot\mathbf{q}} \ , \quad (12)$$

and comparing Eqs.(11) and (8), it is clear that in the limit $\beta\to\infty$ we may identify [17]

$$S^{\alpha\alpha}(0,0) = -\Lambda_1(\mathbf{k}^\alpha, 0; 0, 0) \equiv -\Lambda_1(0) \ . \qquad (13)$$

Thus, Eq.(10) amounts to an exponentiation of the first non-trivial vertex correction.

Obviously, in the above derivation we have ignored all scattering processes that transfer momenta between different patches on the Fermi surface (the so-called around-the-corner processes). Only then the closed loop theorem [2,3,14] is valid, which implies that the effective interaction in Eq.(8) is simply given by the RPA. The neglect of the around-the-corner processes is one of the main approximations inherent in higher-dimensional bosonization [3,4]. We are not awave of any quantitative calculation of the single-particle Green's function which takes these processes explicitly into account. However, for $q_c \ll \Lambda$ it is reasonable to expect that these processes can indeed be neglected, provided the global topology of the Fermi surface does not play a crucial role. The reason is that for $q_c \ll \Lambda$ the number of around-the-corner scattering processes is always paramatrically smaller than the number of forward scattering processes [3].

Let us now evaluate the above expressions in the case of the singular interaction of the type (1). Previously, Bares and Wen [8] have considered only static properties, such as the momentum distribution. To the best of our knowledge, a calculation of dynamic properties, such as the frequency-dependent density of states $\rho(\omega)$, cannot be found in the literature. Such a calculation is straightforward within the framework of higher-dimensional bosonization. The calculations are simplified if one uses the fact that the non-Fermi liquid behavior is due to the existence of a collective plasmon mode $\omega_{\mathbf{q}}$ in the density-density correlation function. The long-wavelength dispersion of this mode is [3,18]

$$\omega_{\mathbf{q}} \sim \frac{v_F \kappa}{\sqrt{d}} \left(\frac{|\mathbf{q}|}{\kappa} \right)^{1-\eta/2} \ , \qquad (14)$$

where $\kappa = (g_c^2 \nu)^{1/\eta}$, and ν is the non-interacting density of states. It turns out that this mode completely determines the infrared behavior of the single particle Green's function. Because for small wave-vectors $\omega_{\mathbf{q}} \gg v_F|\mathbf{q}|$, we may restrict the frequency integration in Eq.(8) to the regime $|\omega_m| \gg v_F|\mathbf{q}|$. Evidently, in this regime the terms $\mathbf{v}^\alpha \cdot \mathbf{q}$ in the denominator of Eq.(8) can be ignored, which implies that the *leading long-distance or long-time behavior of the Debye-Waller factor $Q^{\alpha\alpha'}(\mathbf{r},\tau)$ does not depend on the patch indices α and α'*. Furthermore, the RPA interaction $f^{\rm RPA}(\mathbf{q}, i\omega_m)$ can be replaced by its limit for $|\omega_m| \gg v_F|\mathbf{q}|$, which is given by

$$f^{\rm RPA}(\mathbf{q}, i\omega_m) \approx \frac{1}{\nu} \left(\frac{\kappa}{|\mathbf{q}|} \right)^\eta \frac{\omega_m^2}{\omega_m^2 + \omega_{\mathbf{q}}^2} e^{-|\mathbf{q}|/q_c} \ . \qquad (15)$$

After some straightforward manipulations we find that in the limit $\mathcal{V} \to \infty$ and $\beta \to \infty$ the dominant contribution to the Debye-Waller-factor can be written as

$$Q^{\alpha\alpha'}(\mathbf{r},\tau) \approx -\frac{\sqrt{d}}{2} \frac{\kappa^{\frac{\eta}{2}}}{k_F^{d-1}} \frac{\Omega_{d-1}}{\Omega_d} \int_0^\pi d\vartheta \sin^{d-2}\vartheta$$
$$\times \int_0^\infty dq \ q^{d-2-\frac{\eta}{2}} e^{-\frac{q}{2q_c}} \left[1 - e^{-\omega_{\mathbf{q}}|\tau|} \cos\left(q|\mathbf{r}|\cos\vartheta\right) \right] \ . \qquad (16)$$

Here Ω_d is the surface area of a d-dimensional unit sphere, and we have used $\nu = (\Omega_d/(2\pi)^d) m k_F^{d-2}$.

Let us now focus on the case $\eta = 2(d-1)$. As mentioned above, in this case perturbation theory is plagued by logarithmic singularities. The long-distance behavior of $Q^{\alpha\alpha'}(\mathbf{r},0)$ can be easily extracted, and we obtain for $|\mathbf{r}| \to \infty$

$$e^{Q^{\alpha\alpha'}(\mathbf{r},0)} \sim (q_c|\mathbf{r}|)^{-\gamma} \ , \qquad (17)$$

where the anomalous dimension is given by

$$\gamma = \frac{\sqrt{d}}{2} \left(\frac{\kappa}{k_F} \right)^{d-1} = \left(\frac{dmg_c^2}{2^{d+1}\pi^{\frac{d}{2}}\Gamma(\frac{d}{2})k_F^d} \right)^{\frac{1}{2}} \ . \qquad (18)$$

From this we can obtain the momentum distribution $n^\alpha(\mathbf{q}) = n(\mathbf{k}^\alpha + \mathbf{q})$ near the non-interacting Fermi surface. For $\gamma < 1$ we find

$$n^\alpha(\mathbf{q}) = \frac{1}{2} \left[1 - {\rm sgn}(q_\parallel^\alpha) \Gamma(1-\gamma) \frac{\sin(\frac{\pi}{2}\gamma)}{\frac{\pi}{2}\gamma} \left| \frac{q_\parallel^\alpha}{q_c} \right|^\gamma \right] \ , \qquad (19)$$

where $q_\parallel^\alpha$ is the projection of $\mathbf{q}$ onto $\mathbf{k}^\alpha$. Similarly we can compute $Q^{\alpha\alpha'}(0,\tau)$ for $\eta = 2(d-1)$. We find for $\tau \to \infty$

$$e^{Q^{\alpha\alpha'}(0,\tau)} \sim \left(\frac{\kappa}{q_c} \right)^\gamma (\kappa v_F|\tau|)^{-\tilde{\gamma}} \ . \qquad (20)$$

where for $1 \le d < 2$ the *dynamic anomalous dimension* $\tilde{\gamma}$ is

$$\tilde{\gamma} = \frac{\gamma}{2-d} \quad . \qquad (21)$$

Recall that we have excluded the case $d = 2$, which for $\eta = 2(d-1)$ corresponds to $\eta = 2$ [9]. The small frequency behavior of the density of states can now be calculated, and we finally obtain

$$\rho(\omega) = -\frac{1}{\pi} {\rm Im G}(\mathbf{r} = 0, \omega + {\rm i}0^+) \propto |\omega|^{\tilde{\gamma}} \ . \qquad (22)$$

We conclude that in dimensions $1 < d < 2$ the singularities in the momentum distribution and in the density of states of our higher-dimensional Luttinger liquid are characterized by *different anomalous dimensions*. Although we have shown this only for a specific class of models, we believe that this is a general property of higher-dimensional Luttinger liquids. The reason is that in $d > 1$ the Fermi surface consists of a $d-1$-dimensional continuum, so that loop integrations will always involve some kind of angular averaging over the Fermi surface. This destroys the symmetry between space and time variables, which is only preserved in $d = 1$.

III. THE POLARIZATION

Let us now study how singular interactions of the type (1) affect the low-frequency behavior of the irreducible polarization $\Pi_\star(\mathbf{q},\omega)$. First of all, let us recall that for small momenta $\mathbf{q}$ and small frequencies ω the leading singular corrections to $\Pi_\star(\mathbf{q},\omega)$ cancel, so that

$$\Pi_\star(\mathbf{q},\omega) \approx \Pi_0(\mathbf{q},\omega) \ , \ |\mathbf{q}| \ll k_F \ , \ |\omega| \ll E_F \ , \qquad (23)$$

where Π_0 is the non-interacting polarization. Eq.(23) is a consequence of a complete cancellation of the most singular self-energy and vertex corrections, which happens not only in $d = 1$ [14] but also in higher dimensions [2,3,5]. For $|\mathbf{q}| = O(k_F)$ this cancellation does in general not occur. In fact, in the one-dimensional Tomonaga-Luttinger model it is well known that for momentum transfers $|\mathbf{q}| \approx 2k_F$ the irreducible polarization (as well as various other two-particle correlation functions) exhibits anomalous scaling behavior, very much like the single particle Green's function, although with different anomalous exponents [15]. We now show that this property of one-dimensional Luttinger liquids does in general not survive in higher dimensions. We would like to emphasize that the interaction of the type (1) has no momentum transfers of order $O(k_F)$, so that bosonization can be used to sum the dominant singularities. It is the *external* momentum transfer of the irreducible polarization which is approximately $2k_F$.

Within the same approximations as described in Sec.II, the real-space, imaginary-time Fourier transform of $\Pi_\star(\mathbf{q},i\omega_m)$ can be written as

$$\Pi_\star(\mathbf{r}-\mathbf{r}',\tau-\tau') = -\sum_{\alpha\alpha'=1}^{M} e^{i(\mathbf{k}^\alpha-\mathbf{k}^{\alpha'})\cdot(\mathbf{r}-\mathbf{r}')} \times \langle \mathcal{G}^\alpha(\mathbf{r},\mathbf{r}',\tau,\tau')\mathcal{G}^{\alpha'}(\mathbf{r}',\mathbf{r},\tau',\tau)\rangle \ . \quad (24)$$

Performing the averaging within the Gaussian approximation, we obtain

$$\Pi_\star(\mathbf{r},\tau) = -\sum_{\alpha\alpha'=1}^{M} e^{i(\mathbf{k}^\alpha-\mathbf{k}^{\alpha'})\cdot\mathbf{r}} G_0^\alpha(\mathbf{r},\tau)G_0^{\alpha'}(-\mathbf{r},-\tau) \times \exp\left[Q^{\alpha\alpha}(\mathbf{r},\tau)+Q^{\alpha'\alpha'}(\mathbf{r},\tau) - Q^{\alpha\alpha'}(\mathbf{r},\tau)-Q^{\alpha'\alpha}(\mathbf{r},\tau)\right] . \quad (25)$$

For $|\mathbf{q}| \ll k_F$ the Fourier transform $\Pi_\star(\mathbf{q},i\omega_m)$ of Eq.(25) is dominated by the diagonal terms $\mathbf{k}^\alpha = \mathbf{k}^{\alpha'}$, and we recover Eq.(23). Similarly, for $|\mathbf{q}-2\mathbf{k}^\alpha| \ll k_F$, Eq.(25) is dominated by $\mathbf{k}^\alpha = -\mathbf{k}^{\alpha'}$, so that the Fourier transform of $\Pi_\star(2\mathbf{k}^\alpha+\mathbf{q},i\omega_m)$ is for $|\mathbf{q}| \ll k_F$ given by

$$\Pi_\star^{2\mathbf{k}^\alpha}(\mathbf{r},\tau) = \Pi_0^{2\mathbf{k}^\alpha}(\mathbf{r},\tau)e^{Q^{2\mathbf{k}^\alpha}(\mathbf{r},\tau)} \ , \qquad (26)$$

where (using the notation $\mathbf{k}^{\bar\alpha} = -\mathbf{k}^\alpha$)

$$\Pi_0^{2\mathbf{k}^\alpha}(\mathbf{r},\tau) = -e^{i2\mathbf{k}^\alpha\cdot\mathbf{r}} G_0^\alpha(\mathbf{r},\tau)G_0^{\bar\alpha}(-\mathbf{r},-\tau) \ , \qquad (27)$$

and the Debye-Waller factor is given by

$$\begin{aligned} &Q^{2\mathbf{k}^\alpha}(\mathbf{r},\tau) \\ &= Q^{\alpha\alpha}(\mathbf{r},\tau)+Q^{\bar\alpha\bar\alpha}(\mathbf{r},\tau)-Q^{\bar\alpha\alpha}(\mathbf{r},\tau)-Q^{\alpha\bar\alpha}(\mathbf{r},\tau) \\ &= \frac{1}{\beta\mathcal{V}}\sum_{\mathbf{q}\omega_m} f^{\mathrm{RPA}}(\mathbf{q},i\omega_m)\frac{4(\mathbf{v}^\alpha\cdot\mathbf{q})^2}{\left[\omega_m^2+(\mathbf{v}^\alpha\cdot\mathbf{q})^2\right]^2} \\ &\quad\times[1-\cos(\mathbf{q}\cdot\mathbf{r}-\omega_m\tau)] \ . \end{aligned} \qquad (28)$$

Note that this Debye-Waller factor is different from the Debye-Waller factor $Q^{\alpha\alpha}(\mathbf{r},\tau)$ of the single-particle Green's function. It should also be mentioned that for any finite number of flat patches the non-interacting polarization $\Pi_0^{2\mathbf{k}^\alpha}(\mathbf{r},\tau)$ exhibits in $d > 1$ logarithmic singularities due to the artificial nesting symmetry, which is generated by replacing the curved Fermi surface by a finite number of flat patches. However, we have isolated possible non-trivial effects due to interactions in the Debye-Waller factor $Q^{2\mathbf{k}^\alpha}(\mathbf{r},\tau)$.

It is not difficult to show that in one dimension Eq.(28) correctly reproduces the irreducible polarization of the Tomonaga-Luttinger model for momenta close to $2k_F$ [15]. Note that in this case the large-distance and long-time decay of $\Pi_\star(\mathbf{r},\tau)$ is characterized by non-universal power laws, analogous to the single-particle Green's function. The anomalous scaling manifests itself via a logarithmic growth of the Debye-Waller factor (28) in the large-distance or long-time limit. The important observation is now that in the higher-dimensional non-Fermi liquid state due to the singular density-density interactions (1) the irreducible polarization $\Pi_\star^{2\mathbf{k}^\alpha}(\mathbf{r},\tau)$ *does not exhibit any anomalous scaling*. The case of the Tomonaga-Luttinger model (corresponding to $d = 1$ and $\eta = 0$) is special, because in this case the collective plasmon mode is linear in $|\mathbf{q}|$, so that it has the same order of magnitude as the single-particle excitation energy of the bare fermions. The fact that in $d > 1$ the leading singularities of $Q^{\alpha\alpha'}(\mathbf{r},\tau)$ cancel in $Q^{2\mathbf{k}^\alpha}(\mathbf{r},\tau)$ follows trivially from the observation that the infrared behavior of the Debye-Waller factor $Q^{\alpha\alpha'}(\mathbf{r},\tau)$ is independent of the patch indices, so that the four terms in the second line of Eq.(28) cancel. As already mentioned in Sec.II, this is due to the fact that for $\eta > 0$ the plasmon energy at long wavelengths is larger than $\mathbf{v}^\alpha\cdot\mathbf{q}$, so that the integrand in Eq.(28) contains an additional small factor of $(\mathbf{v}^\alpha\cdot\mathbf{q}/\omega_\mathbf{q})^2$ as compared with the integrand in Eq.(8). This factor cancels the leading singularity.

It is instructive to relate the constant part of the Debye-Waller factor (28) to vertex corrections, similar to Eq.(13). Consider therefore the leading interaction corrections to the irreducible polarization for momentum transfers close to $2\mathbf{k}^\alpha$ shown in Fig.2. Calculating these corrections in real space and imaginary time, we find that they can be expressed in terms of the Debye-Waller factor $Q^{\alpha\alpha'}(\mathbf{r},\tau)$ defined in Eqs.(7) and (8) as follows,

$$\Pi_{1,\mathrm{self}}^{\alpha\alpha}(\mathbf{r},\tau) = 2\Pi_0^{\alpha\alpha}(\mathbf{r},\tau)Q^{\alpha\alpha}(\mathbf{r},\tau) \ , \qquad (29)$$

$$\Pi_{1,\mathrm{vertex}}^{\alpha\alpha'}(\mathbf{r},\tau) = -2\Pi_0^{\alpha\alpha'}(\mathbf{r},\tau)Q^{\alpha\alpha'}(\mathbf{r},\tau) \ . \qquad (30)$$

From these expressions it is obvious that an expansion of Eq.(28) to first order in the RPA-interaction exactly reproduces perturbation theory. Not surprisingly, bosonization amounts to an exponentiation of the leading perturbative corrections in real space and imaginary time. Just like in the case of the single-particle Green's function (see Eq.(13)), we can identify the constant part $S^{2\mathbf{k}^\alpha}(0,0)$ of the Debye-Waller factor (28) with a certain combination of vertex corrections. Assuming again that $S^{2\mathbf{k}^\alpha}(0,0)$ is finite, we see that in the limit of large distances or long times interactions simply lead to an overall multiplicative renormalization of the non-interacting polarization by a factor of $e^{S^{2\mathbf{k}^\alpha}(0,0)}$. Note that $S^{2\mathbf{k}^\alpha}(0,0)$ is always greater than zero and is finite for interactions of the type (1) in dimensions $d > 1$. From Eq.(11) it is easy to see that

$$\begin{aligned} S^{\alpha\alpha'}(0,0) &= -\Lambda_1(\mathbf{k}^{\alpha'}, 0; \mathbf{k}^\alpha - \mathbf{k}^{\alpha'}, 0) \\ &\equiv -\Lambda_1(|\mathbf{k}^\alpha - \mathbf{k}^{\alpha'}|) \; . \end{aligned} \tag{31}$$

It follows that $S^{2\mathbf{k}^\alpha}(0,0)$ can also be written as

$$S^{2\mathbf{k}^\alpha}(0,0) = 2[\Lambda_1(2k_F) - \Lambda_1(0)] \; , \tag{32}$$

which should be compared with Eq.(13). From Eq.(32) we can obtain an alternative explanation why for $\eta \geq 2(d-1)$ the non-Fermi liquid nature of the single-particle Green's function does not affect the polarization: although in this case both vertex corrections $\Lambda_1(2k_F)$ and $\Lambda_1(0)$ are divergent, *their difference* $\Lambda_1(2k_F) - \Lambda_1(0)$ *remains finite*. In this respect our higher-dimensional non-Fermi liquid behaves fundamentally different than the one-dimensional Tomonaga-Luttinger model, where $\Lambda_1(2k_F)$, $\Lambda_1(0)$, *and the difference* $\Lambda_1(2k_F) - \Lambda_1(0)$ are all singular.

IV. DISORDERED LUTTINGER LIQUIDS IN HIGHER DIMENSIONS

The conventional strategy to study interactions in disordered Fermi systems is to treat first the effect of disorder as accurate as possible, and only after that incorporate the effects of interactions perturbatively. It has recently been pointed out by several authors [12,19] that for strongly interacting Fermi systems this approach may not correctly capture the interplay between disorder and interactions in low dimensions. In fact, in some cases it may be better to solve first the interaction problem without disorder, and then study the additional effect of disorder. Here we would like to point out that in this approach it is crucial to distinguish between the single-particle scattering time $\tau_1(\omega)$ (which is defined in terms of the single-particle Green's function) and the transport scattering time $\tau_{\rm tr}(\omega)$ (which is related to the conductivity). This difference is important *even if the impurity potential is dominated by s-wave scattering.* The reason is that in non-Fermi liquids vertex corrections play a very important role, so that even for a δ-function correlated random potential the low-frequency behavior of $\tau_1(\omega)$ and $\tau_{\rm tr}(\omega)$ can be completely different.

To substantiate this point, let us use the above non-perturbative results of Sec.II and III to study the effect of disorder on the clean system perturbatively. As usual we model the disorder by coupling the charge density to a Gaussian random potential $U(\mathbf{r})$ with zero average and δ-function correlations

$$\overline{U(\mathbf{r})U(\mathbf{r}')} = \gamma_0 \delta(\mathbf{r} - \mathbf{r}') \; , \tag{33}$$

where the overline denotes averaging over the probability distribution of $U(\mathbf{r})$ and the parameter γ_0 is a measure of the strength of the disorder. Within the lowest order Born approximation the disorder gives rise to the following self-energy correction to the single-particle Green's function,

$$\Sigma_1(\omega) = \frac{\gamma_0}{\mathcal{V}} \sum_{\mathbf{k}} G(\mathbf{k}, \omega) = \gamma_0 G(\mathbf{r} = 0, \omega) \; , \tag{34}$$

where G is the Green's function of the interacting many-body system without disorder. The single-particle scattering time $\tau_1(\omega)$ is defined by

$$1/\tau_1(\omega) = -2{\rm Im}\Sigma_1(\omega + i0^+) \; . \tag{35}$$

Using Eq.(22), we see that in our model for $\eta = 2(d-1)$ the single-particle scattering time diverges within lowest order Born approximation as $\tau_1(\omega) \propto |\omega|^{-\tilde{\gamma}}$, where the dynamic anomalous dimension $\tilde{\gamma}$ is given in Eq.(21). As shown in Ref. [12], this result turns out to be only self-consistent for $\tilde{\gamma} > 1/2$. For smaller $\tilde{\gamma}$ this divergence is removed if one uses instead of Eq.(34) the *self-consistent* Born approximation, which amounts to the replacement

$$G(\mathbf{k}, \omega) \to [G^{-1}(\mathbf{k}, \omega) - \Sigma_1(\omega)]^{-1} \tag{36}$$

on the right-hand side of Eq.(34). However, even if $\tau_1(\omega)$ diverges, it would be *incorrect to conclude that the conductivity and the diffusion coefficient diverge as well.* Such a conclusion is only valid as long as vertex corrections are unimportant, which is certainly not the case for higher-dimensional Luttinger liquids. To see this more clearly, one should keep in mind that the Drude formula $\sigma = ne^2\tau_{\rm tr}/m^*$ for the conductivity (n is the density of the electrons and m^* is their effective mass) involves the *transport scattering time* $\tau_{\rm tr}$, which for interacting fermions is in general not identical with the single-particle scattering time τ_1 defined in Eq.(35). Microscopically, the transport scattering time $\tau_{\rm tr}(\omega)$ can be calculated from the imaginary part of the memory function $M(\omega + i0^+)$ [20,21],

$$1/\tau_{\rm tr}(\omega) = {\rm Im}M(\omega + i0^+) \; . \tag{37}$$

For arbitrary complex frequencies z the memory function is defined in terms of the conductivity via

$$\sigma(z) = \frac{ne^2}{m^*} \frac{i}{z + M(z)} \ . \tag{38}$$

Expanding $M(z)$ to first order in the impurity concentration, one obtains in d dimensions [20]

$$M(z) = \frac{\gamma_0}{dn\mathcal{V}} \sum_{\mathbf{q}} \frac{\mathbf{q}^2}{m^*} \frac{\Pi(\mathbf{q}, z) - \Pi(\mathbf{q}, 0)}{z} \ . \tag{39}$$

Thus, to lowest order in γ_0 the transport scattering time is determined by the (reducible) polarization $\Pi(\mathbf{q}, z)$ of the interacting many-body system. Of course, for non-interacting electrons and a δ-function correlated random potential (i.e. s-wave impurity scattering) both definitions (35) and (37) lead to identical results for the scattering times in the limit $\omega \to 0$. However, for the class of higher-dimensional Luttinger liquids studied in this paper the low-frequency behavior of $\tau_1(\omega)$ and $\tau_{\rm tr}(\omega)$ is completely different. This is obvious from the fact that in these systems the polarization does not exhibit any anomalous behavior, so that $\tau_{\rm tr}(0)$ is simply a constant. Thus, to lowest order in the impurity concentration the static conductivity $\sigma(0)$ is finite. Note that this implies also a finite diffusion coefficient $\mathcal{D} = \sigma(0)/(e^2 \partial n/\partial \mu)$, where $\partial n/\partial \mu = \lim_{\mathbf{q} \to 0} \Pi_\star(\mathbf{q}, 0)$ is the compressibility [22]. Hence, within a perturbative approach to lowest order in the impurity concentration there is no sign for a divergent conductivity in our higher-dimensional Luttinger liquids, even in the regime where the single-particle scattering time $\tau_1(\omega)$ diverges for $\omega \to 0$. Of course, the true low-frequency behavior of the conductivity most likely requires a non-perturbative treatment of disorder, and cannot be calculated by simply expanding the memory function to first order in the impurity concentration.

V. SUMMARY

In this work we have used higher-dimensional bosonization to study the correlation functions of a particular class of higher-dimensional non-Fermi liquids, which are characterized by singular density-density interactions diverging in momentum space as $q^{-\eta}$ for small momentum transfers q. We have paid particular attention to the case $\eta = 2(d-1)$ in dimensions $1 \leq d < 2$, which can be considered as a possible generalization of the Tomonaga-Luttinger model to higher dimensions. The main result of this work is the observation that in $d > 1$ not all of the well-known properties of the Tomonaga-Luttinger model survive. Thus, the correlation functions of higher-dimensional Luttinger liquids cannot be obtained by straightforward generalizations of the correlation functions of the one-dimensional models. In particular, we have shown that in $d > 1$ the scaling properties of the single-particle Green's function cannot be characterized by a single anomalous dimension. Furthermore, unlike in $d = 1$, in higher dimensions the polarization for momentum transfers close to $2k_F$ does not show any non-Fermi liquid behavior. Although we have obtained these results for a rather special class of models, we believe that they are quite general and reflect generic properties of higher-dimensional Luttinger liquids. In the language of many-body perturbation theory, the qualitative difference between the low-frequency behavior of the single-particle Green's function and the polarization is due to the importance of vertex corrections, which in $d > 1$ tend to cancel in gauge invariant correlation functions such as the polarization, *even for large momentum transfers*.

We would like to point out that our results have been obtained for singular *density-density interactions*. Recently many authors [10,23] have studied another class of higher-dimensional non-Fermi liquids, where the *current-current interaction* mediated by transverse gauge fields is responsible for the non-Fermi liquid behavior. Our results are not applicable to these more complicated interactions. In contrast to the density-density interactions discussed here, for current-current interactions it is not allowed to ignore the curvature of the Fermi surface in a bosonization approach. Although we know [3,6] how to include curvature effects into the bosonization calculation of the single-particle Greens function, it is not straightforward to generalize this method for the polarization.

Finally, in Sec.IV we have shown that in the perturbative treatment of disorder in higher-dimensional Luttinger liquids it is very important to distinguish between the single-particle scattering time $\tau_1(\omega)$ and the transport scattering time $\tau_{\rm tr}(\omega)$, *even if the impurity potential is dominated by s-wave scattering*. In particular, a possible divergence of $\tau_1(\omega)$ does not necessarily indicate a similar divergence of the conductivity or the diffusion coefficient.

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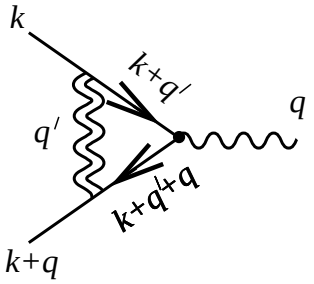

FIG. 1. Leading interaction correction to the density vertex. Here, k is short for $(\mathbf{k}, i\tilde{\omega}_n)$ and q is short for $(\mathbf{q}, i\omega_m)$. The double wavy line is the RPA screened interaction, the wavy line is the external field, and the solid arrows denote non-interacting single-particle Green's functions.

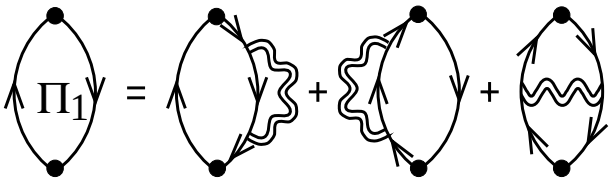

FIG. 2. Leading interaction corrections to the irreducible polarization for momentum transfers close to $2\mathbf{k}^\alpha$. The first two diagrams correspond to self-energy corrections (see Eq. (29)), the third diagram is the leading vertex correction (see Eq. (30)).